Grade Level/Course: 3

Lesson/Unit Plan Name: Area of Complex Figures Foundations

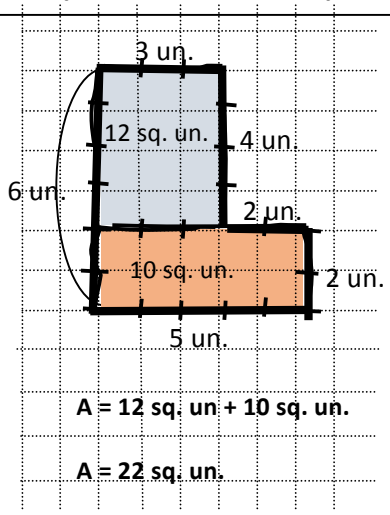
Rationale/Lesson Abstract: Students create and decompose complex figures using **grid paper.** Teachers can use this lesson if they think their class needs foundational work with complex figures *before* moving on to **line drawings**.

Timeframe: 2 -3 days

Common Core Standard(s): 3.MD.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Sample Work from this Lesson:

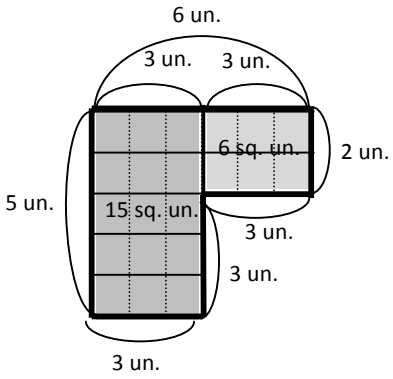
Compose and record complex figures:



A = 12 sq. un + 10 sq. un.

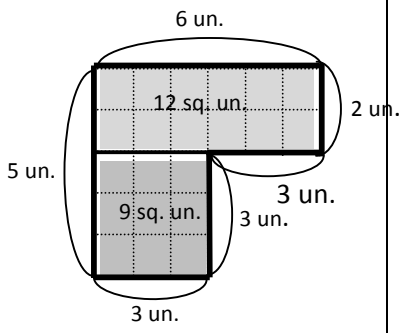
A = 22 sq. un.

Decompose complex figures two ways:



A = 15 sq. un. + 6 sq. un.

A = 21 sq. un.



A = 12 sq. un. + 9 sq. un.

A = 21 sq. un.

Sample from SBAC Practice Test 2013

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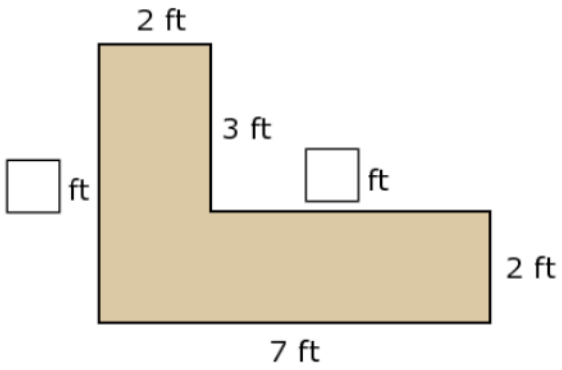
David wants to create the L-shaped desk shown. He decides to buy two rectangular desks and put them together.

- Drag numbers into the boxes to show the missing dimensions.
- Use the Connect Line tool to draw a line dividing the diagram into two desks. Make each desk 5 feet by 2 feet.
- What is the total area of the L-shaped desk? Drag numbers into the box to show your answer.

0 1 2 3 4 5 6 7 8 9

Delete | Add Point | Connect Line

A.



B.

Total area: ft²

Instructional Resources/Materials: Graph paper, scissors, crayons.

Create and Record Complex Figures (1 Day)

We have been working with area of rectangles for a while. Now we are going to work with **complex figures.**

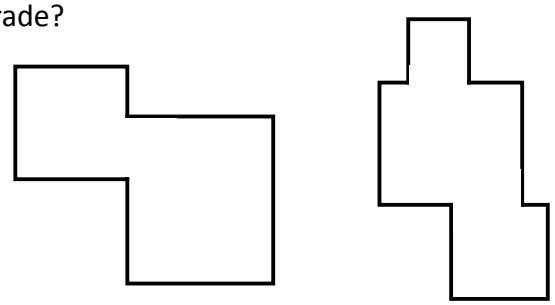
Complex Figure

Complex means many parts.

Figure means shape.

What do you think complex figure means? Share with your neighbor. Quiet hand to share. (A shape with lots of shapes in it.)

The figures you decomposed in the warm up are complex figures. Remember this from first grade?



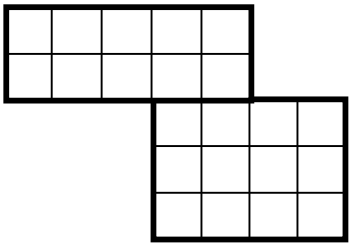
Note: Students decompose figures as early as first grade: 1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

Today we are going to make complex figures. We'll start by cutting out rectangles. There are 3 rectangles on your paper. Cut out all three.

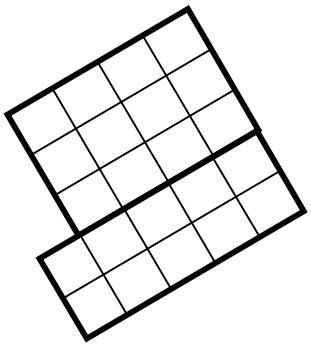
Put the square to the side. That's the bonus shape. We are only going to work with the first two for now.

If I put these 2 rectangles together, I have a complex figure because there are at least 2 parts to it:

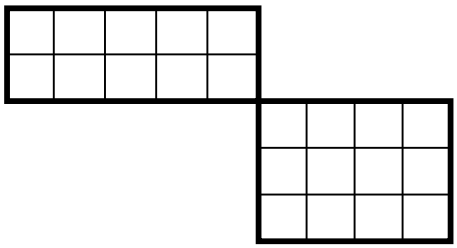
Complex Figure



Complex Figure

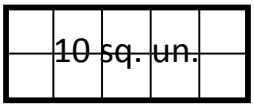


Not a Complex Figure
How is this different?

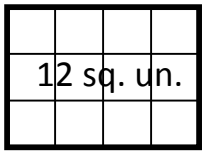


The rectangles need to share at least part of a side. In the last example, they only share a point.

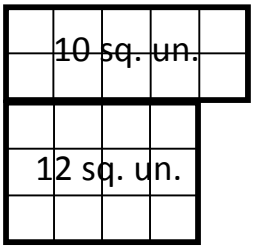
What is the area of this rectangle? (10) 10 what?
10 square units. Let's label that.



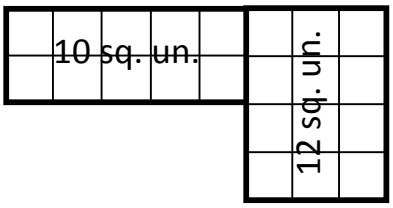
What's the area of this rectangle? (12 square units)
Okay. Label that as well.



If I put these 2 rectangles together,
what would the total area be? (22 square units).



What if I put them together this way?
Is the total area still 22 square units? (yes).
We just made 2 complex figures.
Why is the area the same for both figures?
(They have the same rectangles.)

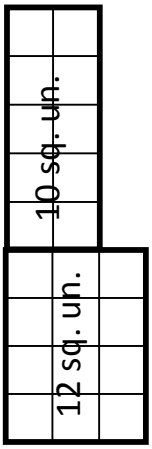


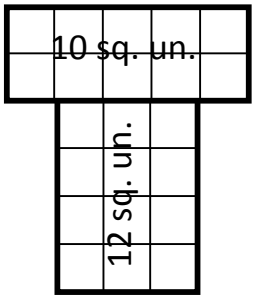
So…no matter how we put these together, we will have an area of 22 square units.

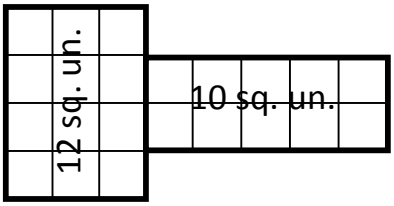
Model a couple other ways to make complex figures.

For the next few minutes I want you work with your neighbor to explore other ways to put these rectangles together to make complex figures.

Circulate to see how students are doing. Students do not glue or tape shapes together; they are only moving them around in different configurations. Here are some ideas:







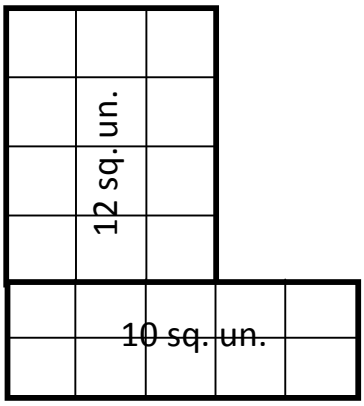
We Do: Record Complex Figure

Now we are going to *record*, or draw some of the complex figures we made.

The graph paper used to record does not need to be the same size as the graph paper the students cut.

Make:

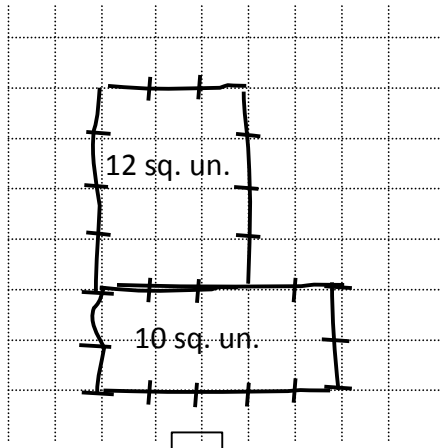
Let's start by arranging our rectangles. Thumbs up when your rectangles are look like this.



Record:

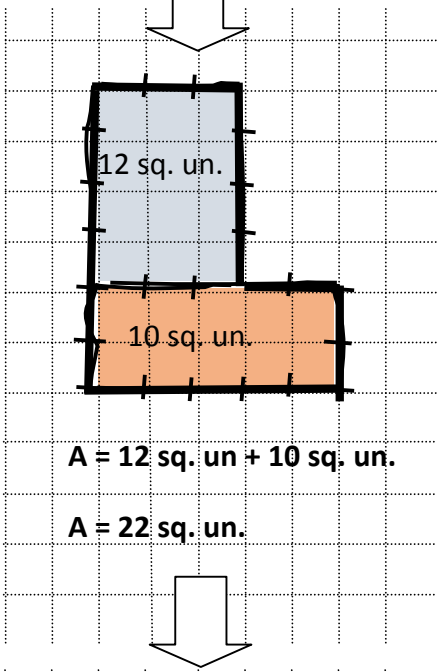
Now we can draw the top rectangle: 4 units down, 3 units across. Then write in the area.

Draw the second rectangle below the first. We already have part of one side drawn for us.



Let's color each rectangle to show the parts of the complex figure. *Students can also outline the perimeter of the whole figure in a darker color. Students may misunderstand and outline each rectangle instead of the just the outside of the figure.*

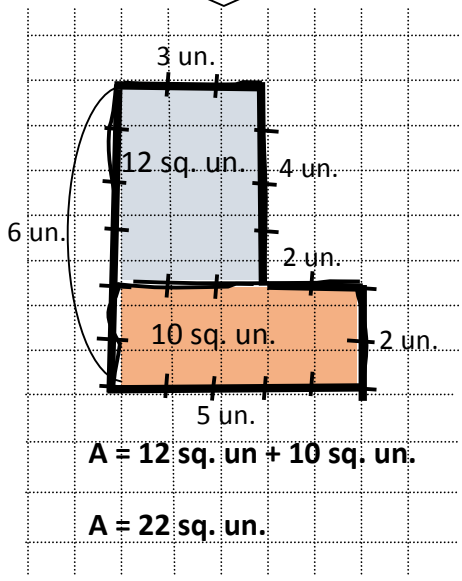
Now show how we got the area: A = 12 square units + 10 square units.



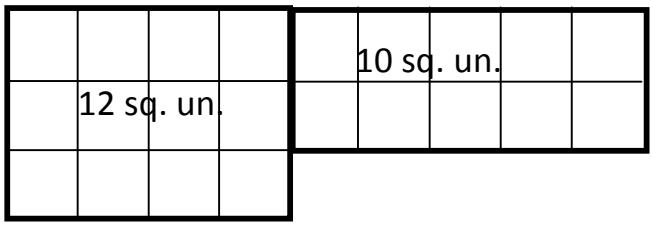
Final student work:

To make our work complete, label the units around the **outside** of the complex figure. (How long is this side?...)

If students are inundated at this point, it's okay to skip writing the units around the figure, as long as it's addressed at a later time.



We Do: *(In this example, move students toward independence. Reference the previous example to help them remember.)*

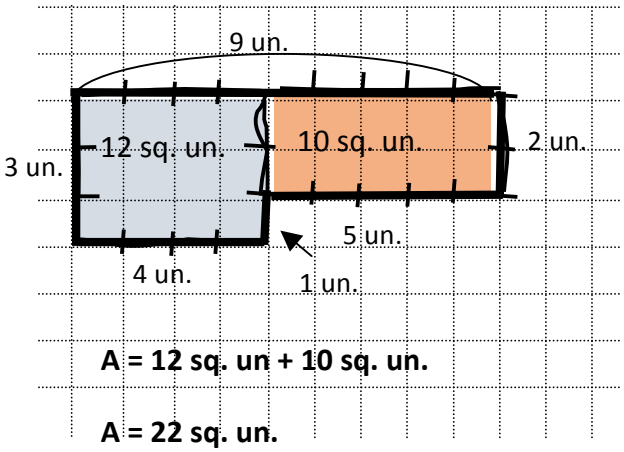


One Way to Coach them to Independence:

Raise your hand if you think you can do most of this on your own.

We'll do one more together, and then one all on your own.

Make your rectangles look like this. Thumbs up when you're done.



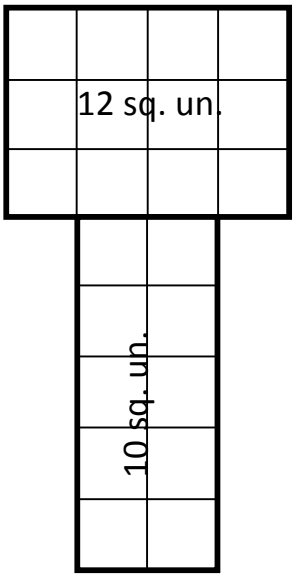
How are you going to record this complex figure? What will you draw first? Think. Share with your neighbor. Quiet hand to share. (Draw one rectangle, then the second.) Can you draw these rectangles to look like this? Can you label the area of each rectangle? Thumbs up when you are done. *Give students time to draw.*

Now what do we get to color? (Color each rectangle.) What do we outline? (the outside of the shape) *Give students a few minutes for that.*

Look at our last example. How did we show the total area? (We wrote the equation.) Then what? (We wrote the units around the outside of the shape.) *Give them a few minutes for that.*

Show student sample to class.

You Try:

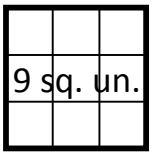


Raise your hand if you could do the next one all on your own. Raise your hand if you might need help from a partner.

Can you draw and label each rectangle? Can you color your drawing like we did before? Can you show how you got the area? Can you label the units on the outside?

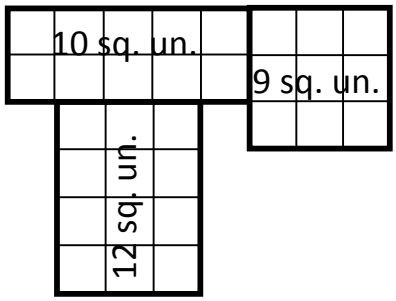
Once students show that they can do the task independently, they can make their own complex figures using the 2 shapes, and record them on their graph paper. If you think they are ready, you can do the extension below.

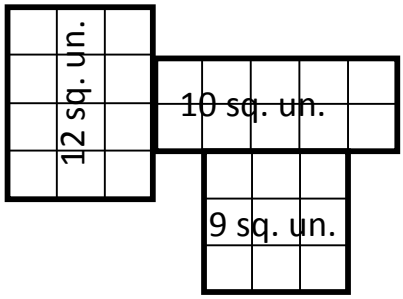
(Extension) If you've already made one on your own, you can add the bonus shape (the square) to your next complex figure if you want.

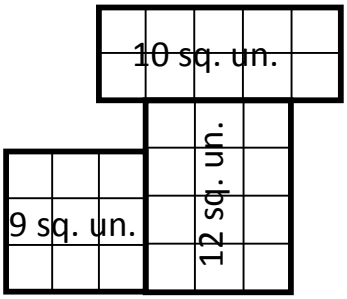


Show a few examples. Remind them that their area will be different because we have a new part. At this point, focus is meant to be on exploring different configurations, not on labeling the perimeter.

Possible configurations with the bonus square.







At the end of the session, you can put student work on the document reader, or have the students do a gallery walk.

Sum it up: Thumbs up if you learned something new today. Share with your neighbor your favorite part of the lesson.

The following can be an exit ticket or a warm up question for the next day.

Key: A. Area = 24 sq. un. ; B. not a complex figure; C. Area = 38 sq. un.

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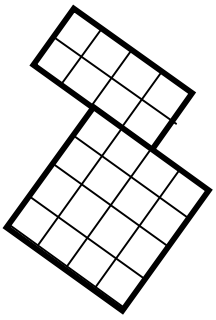
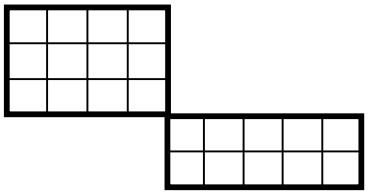
1) Decide which of the following is NOT a complex figure. Cross it out.
2) Find the area of each complex figure. Then write the units around the outside of each complex figure.

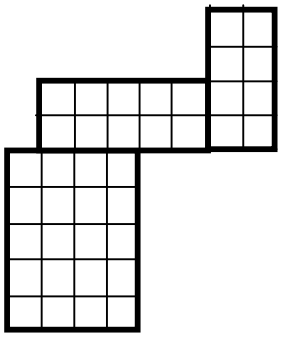
A.

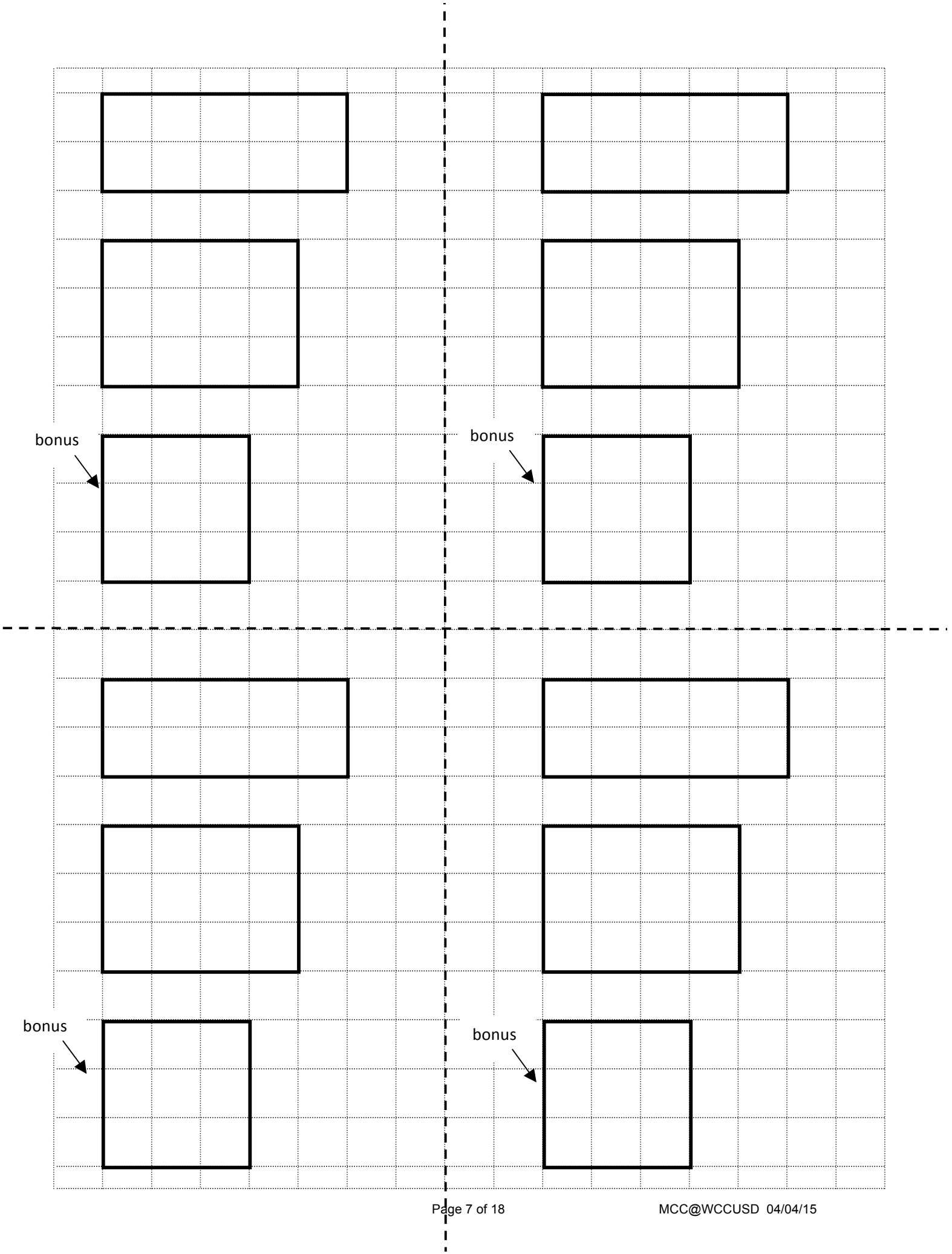


B.



C.

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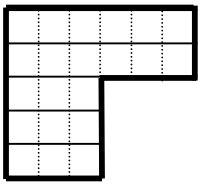
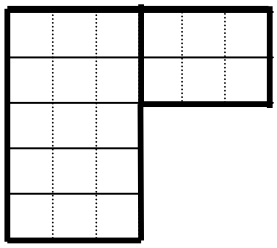
Decompose Complex Figures Two Ways (1 Day)

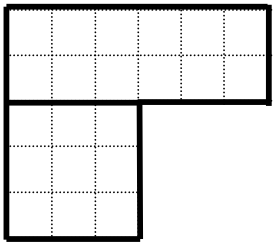
Yesterday we made complex figures. Now we are going to break them apart, or decompose them to find the area.



We need to decompose this complex figure into 2 rectangles.
Share with your neighbor how you would decompose the figure. *Share out.*

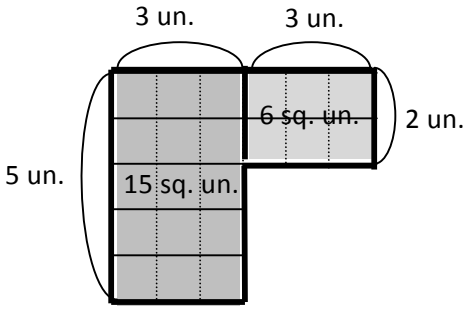
So there are 2 ways…with a vertical line or a horizontal line.





We Do: What are the sides of this rectangle? (3 units and 5 un.) What is the multiplication fact we could use? (3 x 5 = 15). *Repeat for other rectangle. Students can also color in each rectangle a different color. Please note the arcs can be used to help student understanding, but they are not necessary.*

Now find the total area using the same type of equation as yesterday.



A = 15 sq. un. + 6 sq. un.

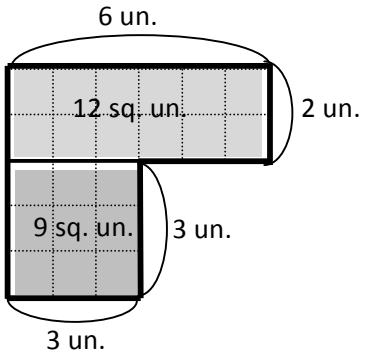
A = 21 sq. un.

You Try: Can you…

…label the sides of each rectangle?

…find the area of each rectangle and add them up?

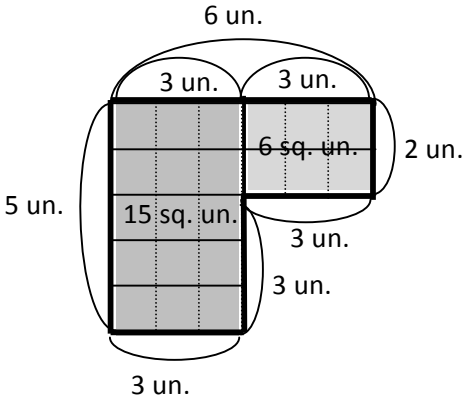
…color each rectangle a different color?



A = 12 sq. un. + 9 sq. un.

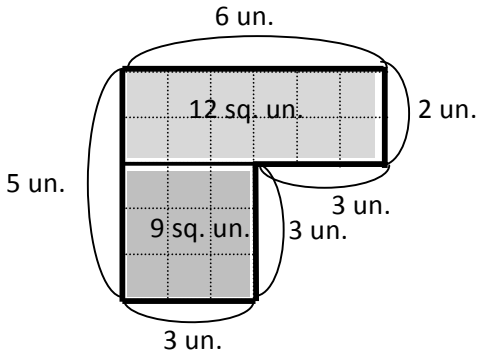
A = 21 sq. un.

To finish off our work, label the rest of the units on the outside of the figure. *As noted before, you can use the arcs as needed.*



A = 15 sq. un. + 6 sq. un.

A = 21 sq. un.



A = 12 sq. un. + 9 sq. un.

A = 21 sq. un.

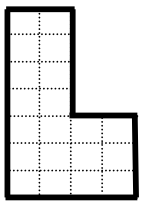
Extension: Find the perimeter.

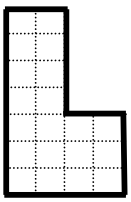
P = 5 un. + 6 un. + 2 un. + 3 un. + 3 un. + 3 un.

P = 11 un + 5 un. + 6 un.

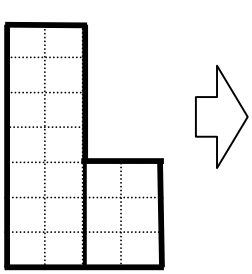
P = 22 units

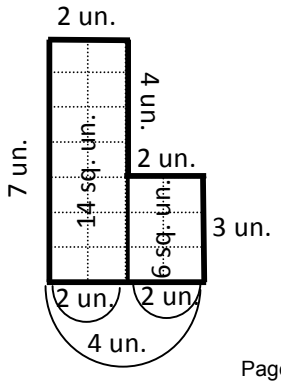
Look at the next two figures. Decompose each figure into different rectangles.





You Try: Find the area of the figure that's decomposed this way:





Thumbs up if you know how to do that. When you are done with the area, what do you do? (label the units on the outside)

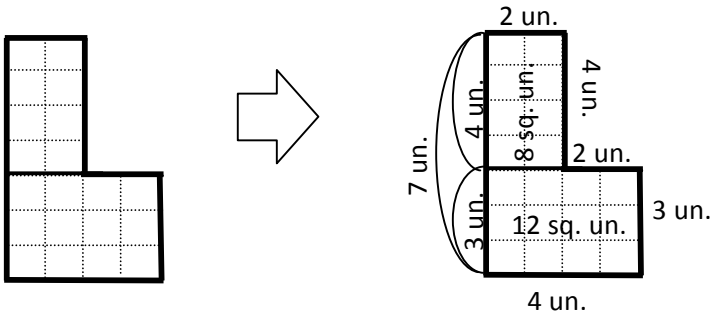
Key:

A = 14 sq. un. + 6 sq. un.

A = 20 sq. un.

Extension: P = 22 un.

You Try: Do the same for the second figure.



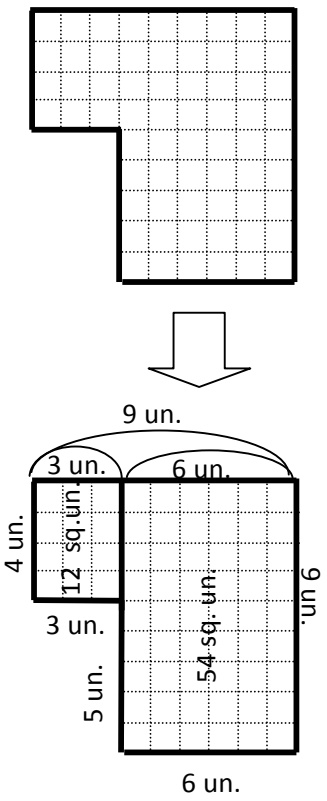
Key:

A = 8 sq. un. + 12 sq. un.
A = 20 sq. un.

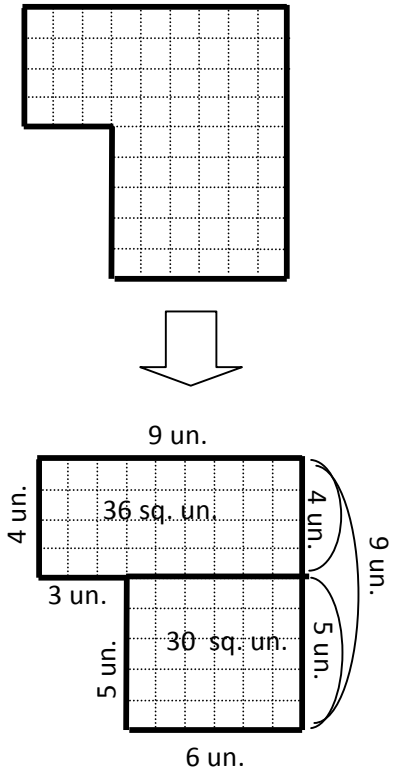
Did everyone get 20 square units for the second figure? Good.

Which way of decomposing was easier? The first way or the second way? Share with your neighbor.

You Try: Decompose each figure. Find the area. Label all the units around the outside of the figure. Check your work with your neighbor.



A = 12 sq. un. + 54 sq. un.
A = 66 sq. un.



A = 36 sq. un. + 30 sq. un.
A = 66 sq. un.

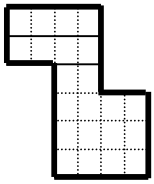
Which way was more efficient? Discuss with your neighbor. *(Share out.)*

Show how you are doing. Show me 3 fingers if you feel you really have it, 2 fingers if you kind of get it, and 1 finger if you are lost.

If students are doing pretty well, you can move on to more complicated figures. This can be done with some guidance or as independent exploration.

Now that we've done the work we need to for today, let's stretch our brains a little. We are going to decompose these types of figures. How many rectangles do you think we can decompose these figures into? Share with your neighbor. Show me on your fingers how many.

Now, we are going to try to decompose figures like this into 3 rectangles.

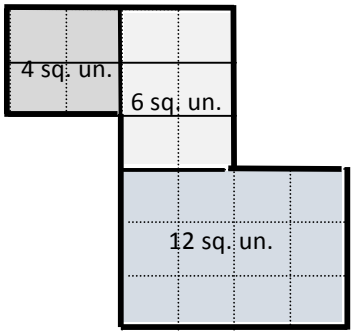


Thumbs up if you are ready for the challenge.

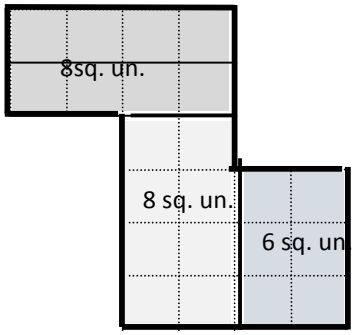
It's okay if you don't get it right the first time. This is bonus work.

If you finish, decompose it a second way. Remember to check your work with your neighbor, and be sure you get the same area both times.

Possible ways:

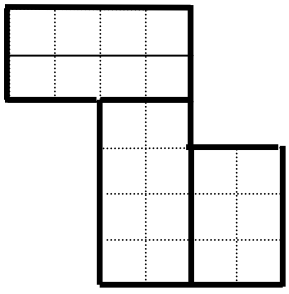


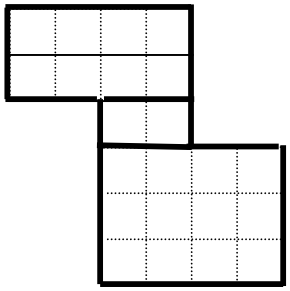
A = 4 sq. un. + 6 sq. un. + 12 sq. un.
A = 10 sq. un. + 12 sq. un.
A = 22 sq. un.



A = 8 sq. un. + 8 sq. un. + 6 sq. un.
A = 16 sq. un. + 6 sq. un.
A = 22 sq. un.

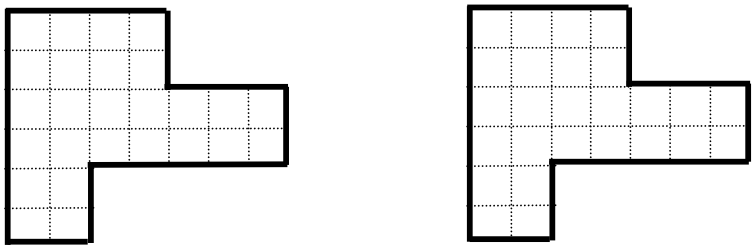
Other Ways to Decompose:



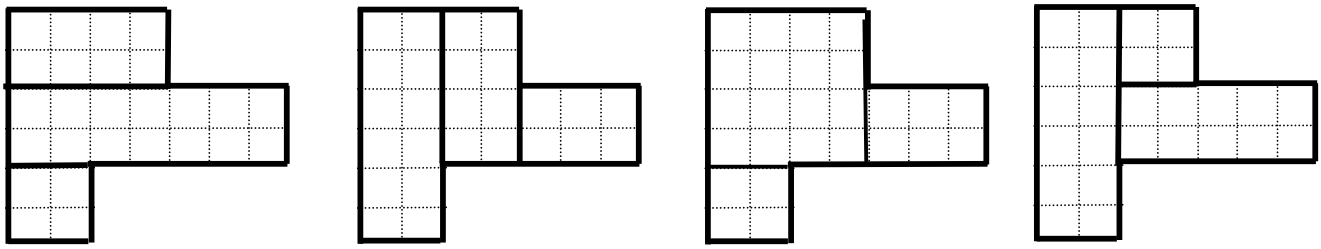


If student decompose into 4 rectangles, that's fine too. What's important is that they are seeing the different ways to decompose the figures.

You Try:

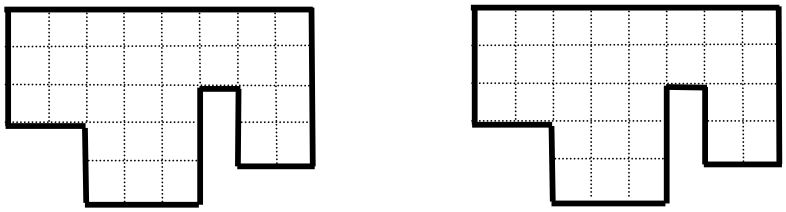


Possible ways to decompose: Total Area is 26 sq. un.

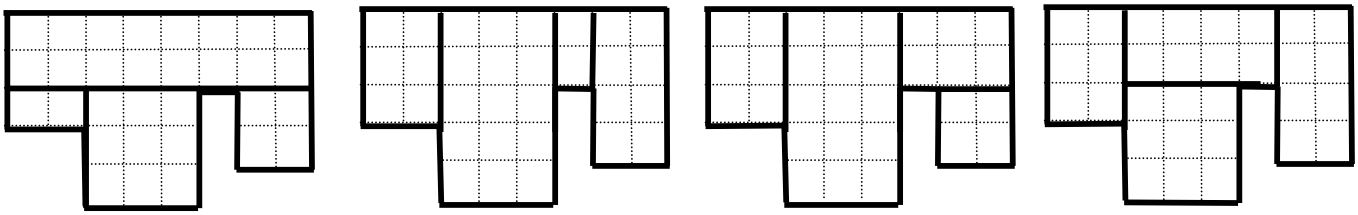


Super Challenge You Try: Decompose two ways. Try to find 4 or 5 rectangles.

Key: A = 31 sq. un.



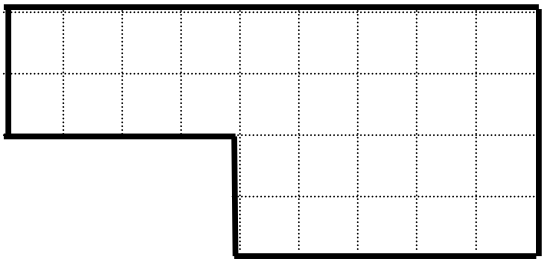
Possible ways to decompose:



Exit Card: Key: Area = 28 sq. un.

Find the area.

Label the units on the outside of the figure.

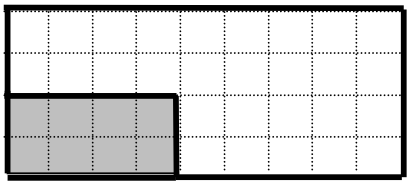


Note to teachers: The standard 3.MD.7d focus on area being **additive.** However, if you want to offer another extension for your students, you can look at the **subtractive** nature of area.

For example, to find the area of the figure used in the Exit Ticket:

Find the area of the large rectangle.
4 un × 9 un. = 36 sq. un.

Find the area of the rectangle formed by the figure and the large rectangle.
2 un. × 4 un. = 8 sq. un.

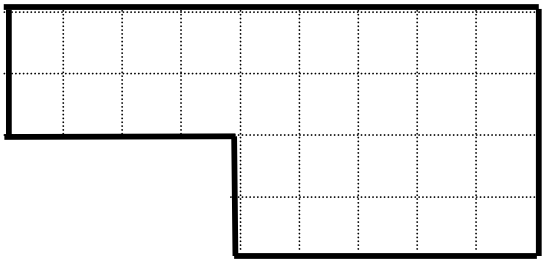


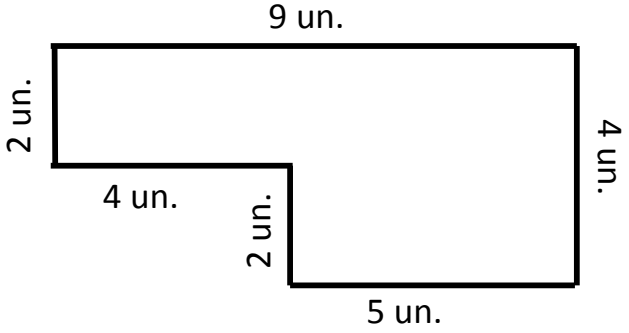
Subtract the small rectangle from the large rectangle.

36 sq. un. - 8 sq. un. = **28 sq. un.**

Next Steps: Transition to line drawings.

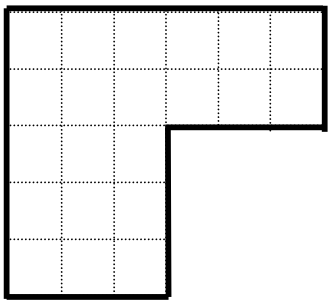
What is the same or different about these 2 figures? (They both have the same units. They are the same shape. One has a grid and the other does not.)

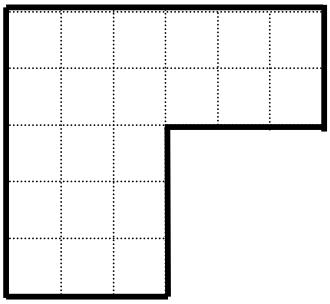


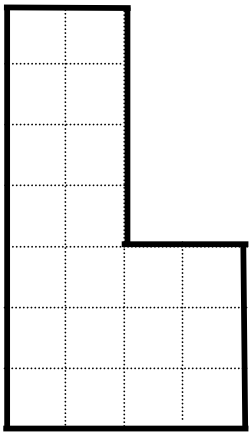


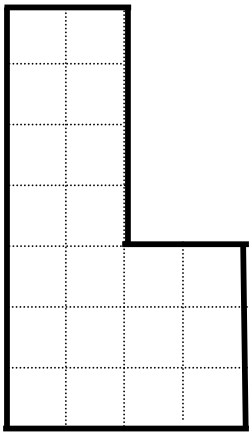
Let's find the area for each figure. *(Decompose each figure the same way.)* What is easier about the new way? What do you have to be careful of?

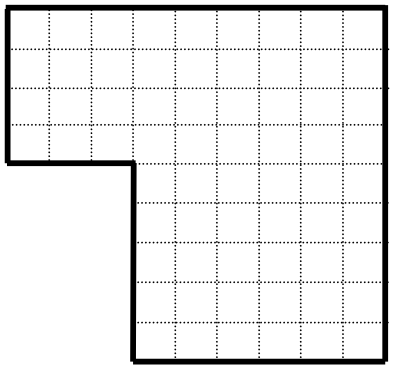
Students move to working with just line drawings. Incorporate customary and metric units they are familiar with: inches, feet, centimeters, and meters.

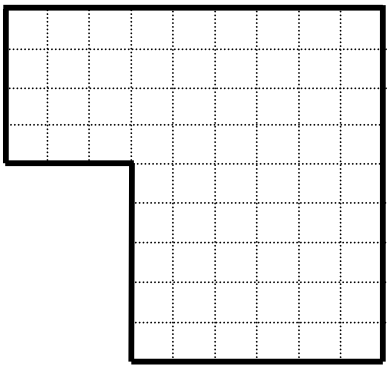


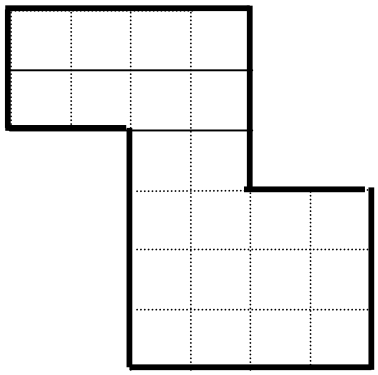


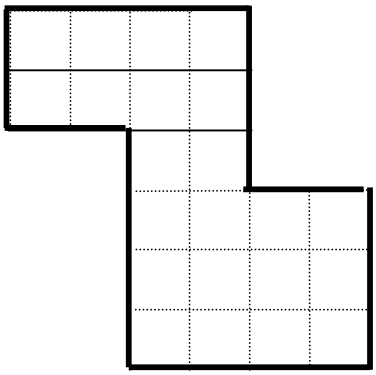


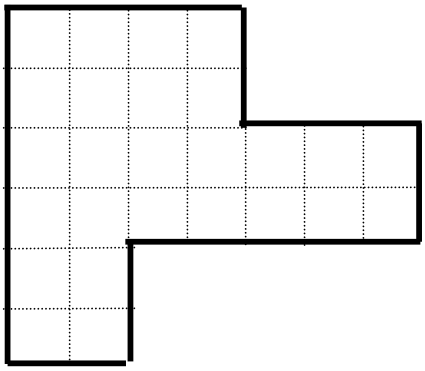


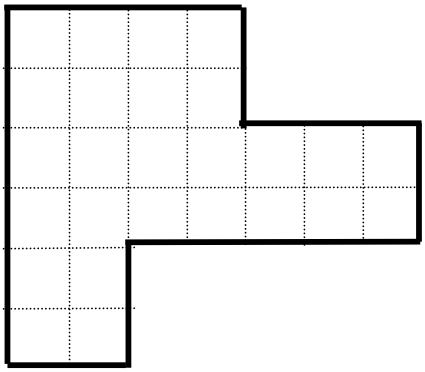


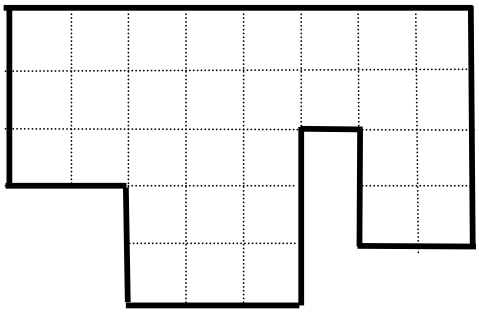


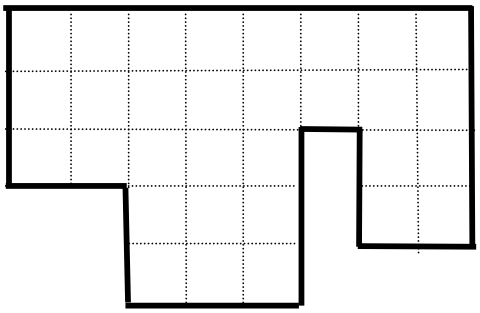












Warm-Up

Test Practice: Mixed Review

True or False:

572 is closer to 570 than 580. ◯ True ◯ False

800 + 50 + 6 = 700 + 140 + 16 ◯ True ◯ False

All rectangles have 2 long sides and 2 short sides. ◯ True ◯ False

Review: CCSS 3.OA.7

Choose one problem. Find the quotient using divvy out with numbers.

Choice A: 72 ÷ 8

Choice B: 84 ÷ 3

Choice C: 145 ÷ 5

Current: CCSS 3.MD.7b

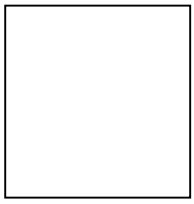
Find the **area** for the rectangle.
Show your work.

4 ft.

9 ft.

Find the **area** for the square.
Show your work.

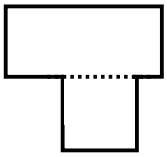
5 cm.



Other:

Decompose each figure into rectangles, or squares.

Example:



or

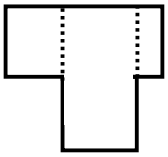


Figure 1:

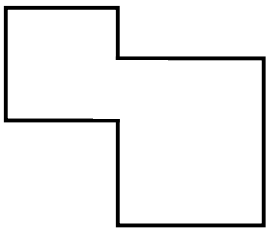
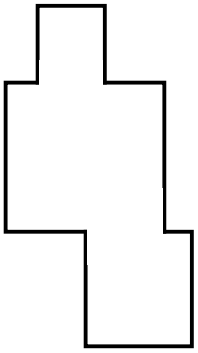


Figure 2:



Warm Up *Key*

Quad II:

True or False:

572 is closer to 570 than 580. ● True ○ False

800 + 50 + 6 = 700 + 140 + 16 ● True ○ False

All rectangles have 2 long sides and 2 short sides. ○ True ● False

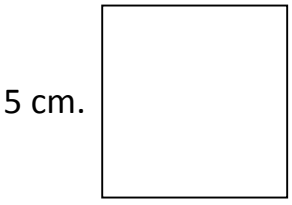
Quad III:

Find the **area** for the rectangle.

A = 4 ft. × 9 ft.

A= 36 sq. ft.

Find the **area** for the square.



A = 5 cm. × 5 cm.

A= 25 sq. cm.

Quad I:

Choose one problem. Find the quotient using divvy out with numbers.

Choice A: 72 ÷ 8 = 9

Choice B: 84 ÷ 3 = 28

Choice C: 145 ÷ 5 = 29

The idea here is that students can choose a problem appropriate for their comfort level. Students who need to work on basic facts can choose A. Others ready for a challenge can choose B or C. There are different ways for students to solve. Examples:

5 2 2	5 2 2	5 2 2	5 2 2	5 2 2	5 2 2	5 2 2	5 2 2	40
9	9	9	9	9	9	9	9	56
								72

5 4	5 4	5 4	5 4
9	9	9	9
5 4	5 4	5 4	5 4
9	9	9	9

$$\begin{array}{r} 72 \\ -\,40 \\ \hline 32 \\ -\,32 \\ \hline 0 \end{array}$$

Quad IV: Possible Answers

Figure 1:

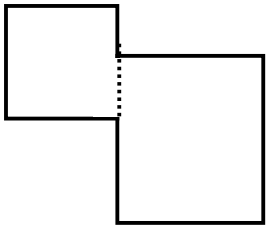


Figure 2: